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Do small systems equilibrate chemically?

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Abstract. Considering particle production in heavy-ion collisions, a particular role has been attributed to strange particles because strangeness was predicted to be a sensitive probe of the properties of QCD matter. The statistical model is very successful in describing the chemical composition of the final state of collisions over a wide range of incident energies. However, without an additional strangeness undersaturation factor, γ_S, hadron gas models hardly reproduce the data from small colliding systems nor from reactions at the smaller collision energies. Here we investigate the influence of an alternative assumption, exact strangeness conservation in small subvolumes of the fireball, on the model predictions. Therefore, we introduce strangeness equilibrated subvolumes. The canonical strangeness suppression in these correlated clusters accounts successfully for the smaller production of strange particles. The system size dependence of the correlation volume and of the thermal parameters are presented.

1 Introduction

Ultrarelativistic nucleus–nucleus collisions are investigated with the goal to study the properties of strongly interacting matter under extreme conditions of high energy density. Hadron multiplicities can provide information on the nature of the medium from which they are originating. The statistical model [1] was recognised as a powerful approach to describe particle yields established at chemical decoupling. In general, this model assumes that at freeze-out the collision fireball appears as a statistical system in thermal and chemical equilibrium. The ensembles are constrained by charge conservation laws.

The statistical model has to be formulated in the canonical ensemble with respect to strangeness conservation if the number of strange particles becomes small. However, in some cases canonical suppression under the assumption of strangeness chemical equilibrium in the whole fireball volume was found to be not sufficient to reproduce the observed yields [2]. It has been proposed to introduce a non-equilibrium factor γ_S in canonical and grand-canonical ensembles as an additional fit parameter to account for the suppressed strange particle phase-space [3]. Here we focus on an alternative approach: the model is extended by correlation volumes which restrict the strangeness chemical equilibrium only to certain subvolumes of the system.

An introduction to the statistical model and its canonical formulation is given in Sect. 2, and also the implementation of clusters is explained. In this work we report on the analysis of experimental data on particle production from p–p and central C–C, Si–Si and Pb–Pb collisions at the top SPS energy. The data selection is demonstrated in Sect. 3, followed by the presentation of the results in Sect. 4. The paper closes with a summary in Sect. 5.

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2 The model description and the basic assumptions

Our approach is introduced in the context of statistical mechanics where the conservation laws are implemented in the grand-canonical and canonical ensembles.

2.1 Grand-canonical ensemble

The statistical model is applicable if the particles observed in high energy collisions are originating from a thermal fireball. This fireball of volume V is characterised by the thermal parameters, the temperature T and baryon chemical potential μ_B, which are the same all over the system. In a grand-canonical ensemble, these parameters fully determine the partition function Z which can be derived from the partition functions Z_i of the particle species i by

$$\ln Z(T,V,\bar{\mu}) = \sum_i \ln Z_i(T,V,\bar{\mu}), \tag{1}$$

with

$$\ln Z_i = \frac{V \cdot g_i}{2\pi^2} \int_0^\infty \pm p^2 \mathrm{d}p \ln\left[1 \pm \exp\left(\frac{\bar{N}_i \bar{\mu} - E_i}{T}\right)\right]. \tag{2}$$

In the model are three chemical potentials involved, $\bar{\mu} = (\mu_B, \mu_Q, \mu_S)$. Only the baryon chemical potential μ_B is an independent parameter while the other two, the charge, μ_Q, and the strangeness chemical potential, μ_S, are constrained by the electric charge of the incoming nuclei and the condition of strangeness neutrality, respectively. In the partition function g_i, the particles degeneracy factor, E_i, its energy, and $\bar{N}$, the quantum numbers enter. The three quantum numbers $\bar{N} = (N_B, N_Q, N_S)$ correspond to the chemical potentials. In order to evaluate Z_i, the integration over the particles momentum p has to be performed. The partition function contains all information to gain access to the densities n_i of all particles species i originating from a thermal source,

$$n_i^{\mathrm{thermal}}(T, \bar{\mu}) = \frac{1}{V} \frac{\partial (T \ln Z_i)}{\partial \mu} . \quad (3)$$

Thermally produced resonances that decay on their path to the detector in the species i contribute to the experimental measured yield and therefore the contributions from all heavier particles j that decay to hadrons i with the branching fraction $\Gamma_{j \to i}$ have to be calculated,

$$n_i^{\mathrm{decay}} = \sum_j \Gamma_{j \to i} n_j \, . \quad (4)$$

The final yield N_i of particle species i is the sum of both contributions, the thermally produced particles and the decay products of resonances,

$$N_i = (n_i^{\mathrm{thermal}} + n_i^{\mathrm{decay}}) V . \quad (5)$$

2.2 Canonical ensemble

In some cases, i.e. if small nuclei collide, if the interactions are peripheral or if the reaction happens at low energies, the number of produced strange particles is small. In general, if the number of charges that have to obey a conservation law is small, the phase-space of this quantity is suppressed. The grand-canonical assumption, requiring that these conservation laws have to be satisfied globally only, does no longer hold. Therefore, here the canonical ensemble is appropriate to describe the hadronic final state of strange particles. The reduction of the density is related to the strangeness quantum number N_S of the species i and can be approximately quantify as

$$n_i^{\mathrm{canon.}} = n_i^{\mathrm{grand\text{-}canon.}} \frac{I_{N_S}(x)}{I_0(x)} , \quad (6)$$

where I_{N_S} and I_0 are the Bessel functions of order N_S and 0, respectively. The argument x represents the sum over all strange hadrons and is in this way related to the volume. As a direct consequence, in canonical ensembles the particle densities, not only their yields, depend on the volume V. Assuming a spherical geometry, the volume can be expressed by its radius R. The canonical suppression for different particle ratios is demonstrated in Fig. 1.

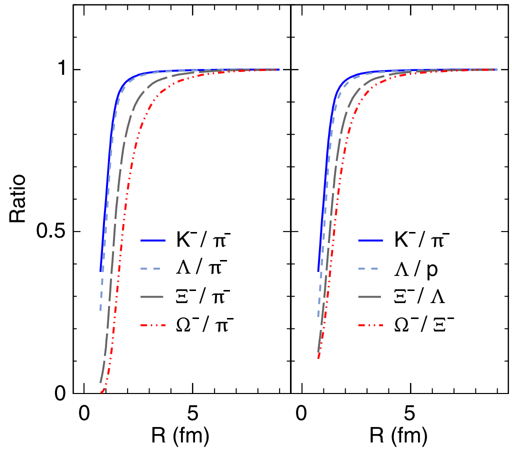

Fig. 1. Different particle ratios as a function of the radius R of a spherical volume. The temperature $T = 170$ MeV and the baryon chemical potential $\mu_B = 255$ MeV were chosen according to the thermal conditions at top SPS energy. All ratios are normalised to the grand-canonical limit in order to demonstrate the effect of canonical suppression

The left hand panel illustrates that the grand-canonical limit is reached, depending on N_S, at radii of a few fm, and that the suppression becomes stronger with increasing strangeness content. On the right hand side, ratios of particles with a difference of 1 in their strangeness quantum numbers are shown: obviously, the suppression of these ratios is similar but not identical; it depends on the strangeness content of both hadrons, not only the difference. The latter is true for the suppression which is caused by the undersaturation parameter γ_S.

2.3 Strangeness conservation within subvolumes

The abundances of strange particles, in particular in the small systems, are found to be below the expectation of the statistical model formulated in the canonical ensemble [2]. Earlier, this want of strangeness in the model was overcome by the arbitrary undersaturation factor γ_S [3]. To cope with the additional suppression, we assume that strangeness is strongly correlated and appears in chemical equilibrium only within subvolumes V_{C} of the system. Consequently, there are two volume parameters in the model. The overall volume of the system, which determines the particle yields at fixed density, and the correlation (cluster) volume V_{C}, which enters through the canonical suppression factor and reduces the densities of strange particles. Assuming spherical geometry, the volume V_{C} is parameterised by the radius R_{C} which serves as a free parameter and defines the range of local equilibrium. A particle with strangeness quantum number N_S can appear anywhere in the volume V; however, it has to be accompanied within the subvolume V_{C} by other particles carrying strangeness $-N_S$ to conserve strangeness exactly.

It should be noted that the canonical description refers to a system in equilibrium, and the reduced phase-space causes a reduction of strange particles. The factor γ_S, however, describes a non-equilibrium situation, yet also reducing strangeness if γ_S is smaller than unity.

As mentioned in the discussion of Fig. 1, right hand side, the suppression pattern for canonical treatment (within clusters) and with γ_S are different. This difference might be used to distinguish between these two approaches.

3 Data sets

In this work we investigate the system size dependence of strangeness production within the statistical model. Experimental results at the top SPS energy, $\sqrt{s_{NN}} = 17.3$ GeV, from p–p and central C–C, Si–Si and Pb–Pb collisions are studied [4–9]. In Figs. 2 and 3 two particle ratios are shown; their evolution with system size is reflected in the thermal parameters, as discussed in Sect. 4.

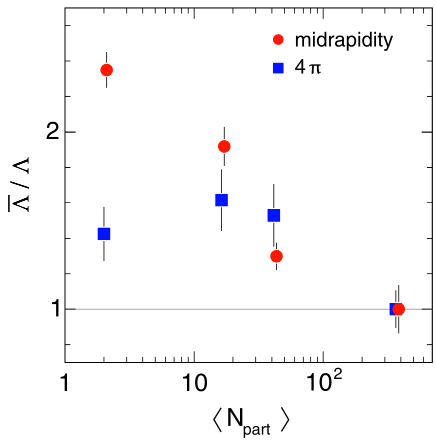

Fig. 2. Midrapidity (*squares*) and 4π (*circles*) $\bar{\Lambda}/\Lambda$ ratio in p–p and central C–C, Si–Si and Pb–Pb collisions, normalised to the Pb–Pb measurement

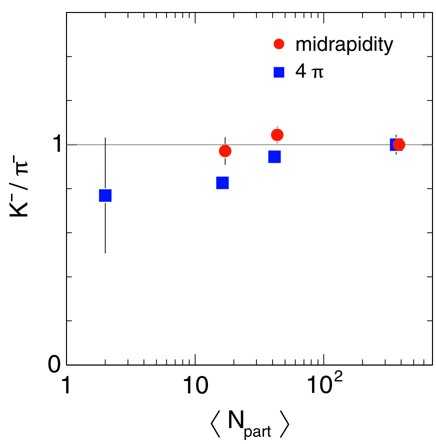

Fig. 3. Midrapidity (*squares*) and 4π (*circles*) K^-/π^- ratio in p–p and central C–C, Si–Si and Pb–Pb collisions, normalised to the Pb–Pb measurement

Since only a limited number of hadrons were analysed in the small systems, we restricted our study to a consistent set of data. The investigated yield and ratios are demonstrated in Fig. 4 for C–C and in Fig. 5 for Pb–Pb collisions.

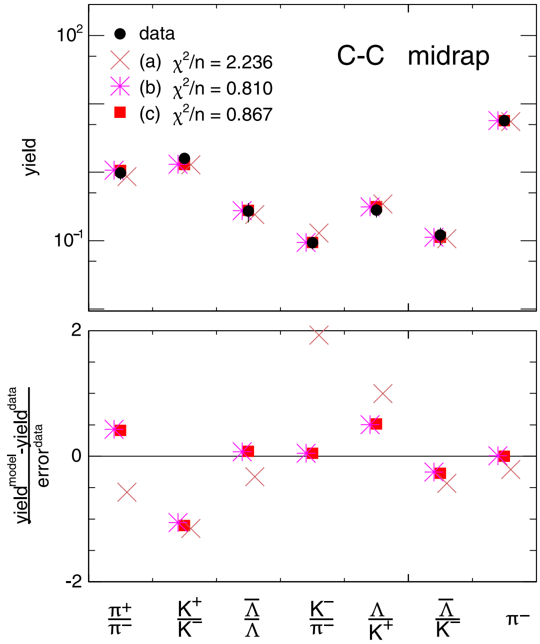

Fig. 4. Midrapidity π^- density and midrapidity particle ratios from C–C collisions at $\sqrt{s_{NN}} = 17.3$ GeV together with model fits (a) (*crosses*), (b) (*stars*) and (c) (*squares*). The *lower panel* shows the deviation of the model fits to data

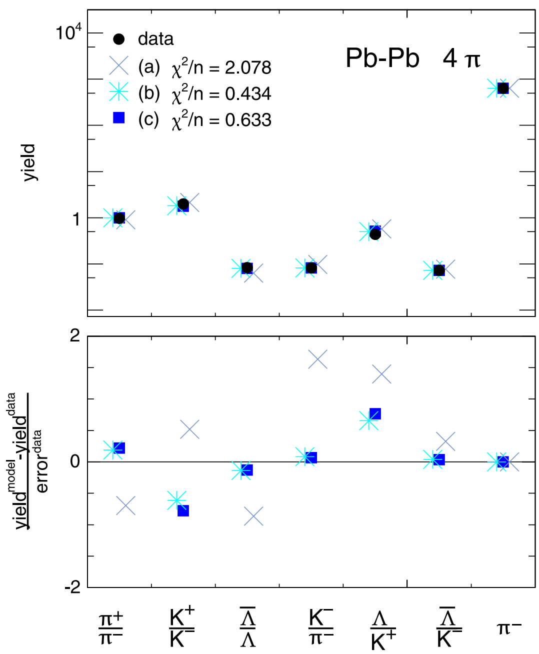

Fig. 5. Integrated π^- yield and integrated particle ratios from Pb–Pb collisions at $\sqrt{s_{NN}} = 17.3$ GeV together with model fits (a) (*crosses*), (b) (*stars*) and (c) (*squares*). The *lower panel* shows the deviation of the model fits to data

Both, midrapidity densities and integrated yields were considered. Only the missing midrapidity data from the p–p interactions make an exception.

4 Results

Firstly, the quality of our model is evaluated by comparing the fit results with the standard methods. Secondly, the results of our model fits are presented and discussed.

4.1 Comparison of various approaches

In Sect. 2.3 the approach of local strangeness conservation within clusters was exposed. Here this concept should be evaluated. Therefore the data sets described in Sect. 3 were fitted with three different model settings.

(a) An equilibrium ansatz, which uses only the temperature T, the baryon chemical potential μ_B and the volume V to describe the data, where canonical suppression affects all hadrons according to the entire fireball volume.
(b) Additionally to (a), the strangeness undersaturation factor γ_S is introduced in the standard way (multiplicative factor for each (anti)strange valence quark).
(c) Additionally to (a), subvolumes with correlated strangeness production are generated by the cluster radius R_{C}.

As examples, Figs. 4 and 5 (upper panels) demonstrate particle ratios measured in C–C and Pb–Pb collisions, respectively, together with model results. Particle ratios from fits with best χ^2 are displayed for all model settings. All statistical model results presented in this paper were achieved with the Thermus package [10]. Option (a) with completely equilibrated strangeness abundance exhibits the largest deviations from the data; the normalised differences are shown in the lower panels. In particular for the small systems, the model setting (a) features dissatisfactorily large values of χ^2 per degree of freedom, as noted in the upper panels. By contrast, both approaches that allow for an extra suppression of the strange particle phase-space, (b) with γ_S and (c) with R_{C}, agree much better with the measurements and yield comparable good descriptions of the data. From the fit quality's point of view, the ansatz with an arbitrary undersaturation factor γ_S and the one with local chemical equilibrium in clusters are on a par. However, the latter can be correlated to properties of the hot and dense medium and systematic studies might provide access to the hadronisation mechanism.

4.2 Cluster results

The statistical model, extended by clusters with local strangeness conservation, model (c), is fitted to all data sets described in Sect. 3. The model results in freeze-out temperatures T which are barely dependent on the system size (see left hand side of Fig. 6). The baryon chemical potential is flat as a function of the system size if total yields are considered. In contrast, μ_B is decreasing towards smaller systems in fits to midrapidity densities (Fig. 6, right hand side). This directly reflects the $\bar{\Lambda}/\Lambda$ ratio which indicates stronger stopping in larger systems (Fig. 2).

The cluster radius, R_{C}, varies between 0.7 fm and 1.4 fm in all data sets under study (Fig. 7). Larger radii in the fits to midrapidity data are correlated with the experimental observation of larger K/π ratios compared to 4π yields (Fig. 3). The increase of strange to non-strange particle ratios by a factor of 2 from the p–p to Pb–Pb reactions is reflected in a variation of the subvolume radius from about 0.7 to 1.4 fm.

In the data sets considered here, the fireball radius R at freeze-out is determined by the multiplicity of π^- mesons. The smaller midrapidity densities cause smaller radii R than the integrated yields. In the comparison of the cluster to the fireball radius it becomes clear that R_{C} has a significantly weaker system size dependence than R (Fig. 8). In the smallest system, this correlation

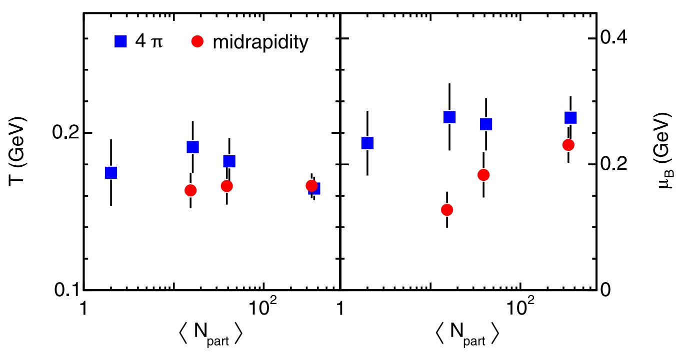

Fig. 6. Chemical freeze-out temperature T (*left*) and baryon chemical potential μ_B (*right*) from fits to midrapidity densities (*circles*) and integrated yields (*squares*) from p–p and central C–C, Si–Si and Pb–Pb collisions, derived with model (c). Please note the suppressed zero in the *left panel*

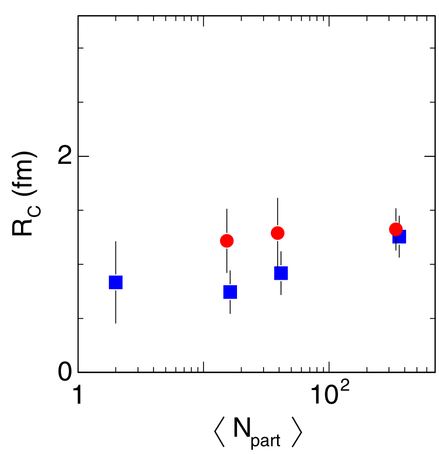

Fig. 7. Cluster radius R_{C} as a function of the number of participants from fits to midrapidity densities (*circles*) and integrated yields (*squares*) from p–p and central C–C, Si–Si and Pb–Pb collisions

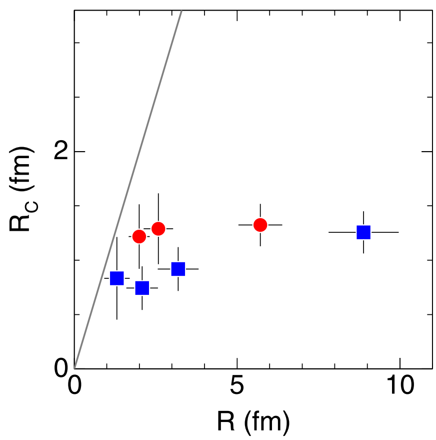

Fig. 8. Cluster radius R_C as a function of the fireball radius R from fits to midrapidity densities (*circles*) and integrated yields (*squares*) from p–p and central C–C, Si–Si and Pb–Pb collisions. The *line* indicates $R_C = R$

length is almost as large as the fireball radius. In contrast, with increasing system size, the clusters with locally conserved strangeness are up to six times smaller than the entire fireballs. In this respect, the fact that R_C is of the order of 1 fm and only weakly increasing with system size could indicate that the $R_C \simeq 1$ fm is the length of the strangeness correlation in a system. By all means, this is an astonishing result and requires further studies. From this analysis we conclude that the transition from suppressed to saturated strangeness production happens in a very narrow range of the correlation length R_C. This behaviour is also known from percolation model calculations [11].

5 Summary

In this paper, a modified version of the statistical model is proposed. The experimentally observed strong suppression of the strange-particle phase-space is retraced by local strangeness conservation within small correlated clusters in the fireball. These subvolumes, in consequence, cause strong canonical suppression and allow one to reproduces successfully the experimental data. Furthermore, the cluster size is found to be weakly depending on the system size. In all data under study, it is of the order of 1 fm, and this coincides with the range of the strong interaction.

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